Dynamic Implications of Subjective Expectations: Evidence from Adult Smokers

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We set up a dynamic discrete choice model with subjective expectations data to explain adult smokers’ smoking decisions. We find important differences between subjective survival probabilities and those estimated using observed mortality data. Subjectively, individuals attach less weight to their health conditions and smoking choices and more weight to such factors as age, race, and parents’ longevity. Moreover, adult smokers are found to care more about their health and to be more forward-looking than predicted by a rational expectations framework. We further show the importance of unobserved heterogeneity in agents’ subjective survival probabilities, and discuss policy implications of subjective expectations. (JEL D12, D84, I12)

Expectations about future events are crucial to decision makers who consider the dynamic implications of their current choices. For example, expectations about future wage differentials affect youth’s schooling and career choices (Dominitz and Manski 1996; Keane and Wolpin 1997); expectations about future income and job security influence people’s consumption and saving patterns (Dominitz and Manski 1997; Manski and Straub 2000); expectations about future Social Security retirement benefits impact labor supply and timing of retirement (Bernheim 1989); and expectations about future morbidity and mortality shape individuals’ health-related behaviors (Viscusi 1990; Gilleskie 1998). How to model these expectations is a fundamental issue in economics. Indeed, because observed choice data alone could be consistent with various combinations of expectations and preferences, model predictions and policy implications depend crucially on the underlying assumptions concerning individual expectations (Manski 2004).

In the existing literature, the standard procedures for recovering individuals’ preferences from observed choices are based upon certain strong assumptions about the formation of subjective expectations, the most common one among which is the rational expectations assumption. This assumption states that agents use all the relevant information when forming expectations about future events, and their expectations do
not systematically differ from the realized outcomes. That is, although the future is not fully predictable, people do not make systematic mistakes when predicting the future, and deviations from actual outcomes are only due to random errors.

In reality, however, individuals’ subjective expectations about the future might be systematically different from those estimated by economists using a rational expectations framework, either because individuals do make systemic mistakes or because individuals have valuable private information concerning their future, which is correlated with their observed states, but cannot be observed by economists. In these cases, assuming rational expectations when individuals actually make their decisions based on their own subjective expectations will result in model misspecification and misleading conclusions about individual preferences, with important implications for economic analyses of decision-making processes and evaluations of public policies.

In this paper, we relax the rational expectations assumption by directly incorporating subjective expectations into a dynamic discrete choice model, to explain how adult smokers decide whether to quit smoking. The goal here is to infer individuals’ utility and time preferences using information on their own smoking decisions and subjective longevity expectations available in the Health and Retirement Study. Specifically, in the first step, we develop a new empirical approach, along the lines of Manski and Molinari (2010) and Hudomiet, Kézdi, and Willis (2011), to analyze subjective longevity expectations data. This approach explicitly models the underlying data-generating process for the true subjective longevity expectations and connects these true expectations with the reported ones using interval responses. Using this approach, we can better address the potential reporting and measurement errors in the survey data, and better deal with the discrepancy in the time horizon between the subjective expectations available in the data and what is needed in the dynamic discrete choice model.

In the second step, we set up a discrete choice model in which adult smokers make the dynamic decisions of whether to quit smoking or not in each period, based on the first-step estimates of their subjective survival probabilities as well as the transitions of their income and health status. We then estimate adult smokers’ utility and time preferences using this subjective dynamic discrete choice model and compare these estimates to those from a rational expectations framework (i.e., no subjective expectations). Finally, to alleviate the potential endogeneity issue, we introduce persistent unobserved heterogeneity in agent’s expectations that cannot be captured by the observable state variables, and discuss its implications for agents’ preferences and choices.

Our empirical results show that adult smokers indeed form their subjective longevity expectations differently from what the rational expectations assumption predicts; and this difference lies mainly in the economic and statistical importance adult smokers attach to various determinants of their survival. For example, objectively, having bad health is the largest threat to one’s survival, having a long-lived parent can only “cancel out” half of the detrimental effect of smoking, and being a smoker is equivalent to being at least four years older in terms of its negative effects on survival. Subjectively, however, having bad health no longer matters the most for one’s survival; it is actually similar in magnitude to having a long-lived parent, which in

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2 Adult smokers in this paper refer to current and former mature smokers. Definitions of these terms are given in Section II.
turn can “cancel out” more than half of the negative effect of smoking. And smoking, subjectively, is only equivalent to ageing for about two years.

Regardless of the source of this discrepancy in the formation of longevity expectations, it results in crucial differences in the estimates of utility and time preferences. For example, in both subjective and rational expectations frameworks, the utility loss from having bad health is greater when people choose to smoke. However, to rationalize the choice of smoking, given its negative health effect, the rational expectations framework requires a much larger gap in utility sensitivity to health status between smoking and not smoking than the model with subjective expectations. We also find that adult smokers are more forward-looking than we would have concluded using the rational expectations framework.

When persistent unobserved heterogeneity in subjective expectations is introduced to our model, private information not captured by the publicly observed information is shown to play an important role. With two types in our empirical setting, we find that individuals are more homogenous within each type. These two types also have different survival probabilities and utility and time preferences. Specifically, when compared to the first type, the second type, which consists of 18 percent of the whole sample, attaches more weight to relatively “exogenous” survival determinants, such as parents’ longevity and age, and less weight to “endogenous” factors, such as smoking and health status. In addition, this second type associates less utility loss with bad health when smoking and is less forward-looking than the first type.

We further consider a counterfactual experiment where adult smokers’ subjective longevity expectations are set to be the same as those implied by the rational expectations assumption. In this case, the average smoking rate would be 8 percentage points lower than the level observed in our sample.

We apply our subjective dynamic discrete choice model to adult smokers’ decisions to quit smoking because smoking is linked to a myriad of quality-of-life reducing health problems such as lung cancer and chronic obstructive pulmonary disease. Actually, tobacco use has been responsible for about one-fifth of the total mortality in the United States since the 1990s, and is therefore considered the number one actual cause of premature deaths and the most important preventable risk to human health. Although smoking decisions are made under considerable uncertainty, with significant pecuniary and nonpecuniary consequences (Sloan et al. 2004), adult smokers’ decisions to quit smoking are determined mainly by longevity and health concerns. The availability of relevant data and the relatively few significant factors in the decision-making process thus make smoking an excellent testing ground for the impact of subjective expectations on individual choices.

This paper makes its main contribution to a growing literature on analyzing individual behavior using subjective expectations data. It has been recognized that combining data on subjective expectations with data on observed choices allows

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4 See, for example, McGinnis and Foege 2004 and Mokdad et al. 2004. In addition, see Chaloupka, Tauras, and Grossman 2000 and Chaloupka and Warner 2000 for excellent reviews of economic studies on smoking.
us to relax certain strong assumptions about the formation of decisionmakers’ expectations. However, few studies have made direct use of subjective expectations data in understanding and predicting choices, with the notable exceptions of Lochner (2007), who investigates the effects of perceptions of the justice system on youth criminal behavior, and Delavande (2008), who analyzes how perceptions about the benefits and costs of different contraceptive methods affect women’s birth control choices. Both studies use static models. To the best of our knowledge, this paper is among the first to use subjective expectations data in a dynamic discrete choice model of (health-related) behaviors, and therefore also contributes to the vast literatures on dynamic discrete choice models and health behaviors.

However, although robust to various specification checks, the specific empirical results of this paper concerning survival probabilities and utility and time preferences need to be taken with caution, as our estimation here is based on a sample of adult smokers only.

The rest of the paper proceeds as follows. Section I sets up a dynamic discrete choice model with subjective expectations. Section II introduces our data. Section III provides details on empirical specification and model estimation, followed by discussions in Section IV of the empirical results and the counterfactual experiment. Finally, Section V concludes.

1. Dynamic Discrete Choice Model with Subjective Expectations

This paper uses a finite-horizon, single-agent, dynamic discrete choice model with subjective expectations. In this class of models, agents choose from a finite set of actions to maximize their expected lifetime utilities, based on their expectations of future state transitions.

This model has the following components:

• A time index, $t \in \{0, 1, 2, \ldots, T\}$.
• A state space, $S$, consisting of both observable and unobservable components. Specifically, $s_t = (x_t, \varepsilon_t) \in S$, where $x_t$ is observable to everyone, while $\varepsilon_t$ is only observable to the agent.
• A choice space, $A$, with a finite number of discrete choices $a_t \in A$.
• Agents’ subjective expectations about future state transitions, $p(s_{t+1}|s_t, a_t)$.
• An exponential discount factor, $\beta \in [0, 1]$.
• An instantaneous period utility function, $u(s_t, a_t)$.

5In addition, complementary to Van der Klaauw and Wolpin (2008), who use subjective expectations as auxiliary data in their structural model, here we treat subjective expectations data as a key component and use them directly in our dynamic discrete choice model. Other recent studies analyzing relationships between subjective expectations and individual behaviors, using more reduced-form models, include Nyarko and Schotter (2002) using experimental data, and Hurd, McFadden, and Gan (1998), Viscusi (1990, 1991), and Khwaja et al. (2009) using survey data on various decisions, such as life-cycle consumption patterns and smoking.

6For detailed reviews of the literature on dynamic discrete choice models, see, for example, Eckstein and Wolpin (1989), Rust (1994), and Ackerberg et al. (2005).

7An alternative approach models individuals as (quasi-)hyperbolic discounters who put less weight on the near future and more weight on the far future, and therefore make time-inconsistent choices. See, for example, Laibson (1997), O’Donoghue and Rabin (1999), Gruber and Köszegi (2001), and Fang and Silverman (2009).
Agents have the following additively separable inter-temporal utility function:

\[ U_t = u(s_t, a_t) + \sum_{\tau=t+1}^{T} \beta^{\tau-t} u(s_\tau, a_\tau), \]

which leads to the following value function at the time of choice:

\[ V(s_t) = \max_{a_t \in A} \left[ u(s_t, a_t) + \int V(s_{t+1}) p(s_{t+1} | s_t, a_t) ds_{t+1} \right]. \]

Intuitively, in this model, agents are faced with a set of discrete choices, each of which is associated with a certain level of lifetime utilities, as shown by equation (1). Because future states are usually uncertain, agents calculate these utilities based on their expectations about the future, given their current choices and states. So, to recover agents’ preferences \( u \) from the observed choices \( a \), one must know their subjective expectations \( p \) about the future states that follow their current choices. Typically in the literature, however, \( p \) is assumed to equal the objective state transitions directly observed in the data. This paper relaxes this rational expectations assumption by incorporating agents’ own subjective expectations about future state transitions into the dynamic decision-making process.

Based on the three assumptions commonly made in the literature concerning the unobservable component in the preferences (Rust 1987), we express the expected maximum value of inter-temporal utilities as the ex ante value function through the following relationship:

\[ V(x_t) = E \max_{a_t \in A} \left[ u(x_t, a_t) + \epsilon_t(a_t) + \beta \int V(x_{t+1}) p(x_{t+1} | x_t, a_t) dx_{t+1} \right], \]

where the part observable to the economist can be collected and termed the choice-specific value function:

\[ V(x_t, a_t) = u(x_t, a_t) + \beta \int V(x_{t+1}) p(x_{t+1} | x_t, a_t) dx_{t+1}. \]

Without loss of generality, consider a two-choice case, where \( A = \{0, 1\} \). The literature (Hotz and Miller 1993) makes it clear that what can be identified from the data is the following difference in the choice-specific value functions:

\[ D(x_t) \equiv V(x_t, 1) - V(x_t, 0) = \ln P(1|x_t) - \ln P(0|x_t). \]

If we further assume that the instantaneous utility functions take on the following linear form:

\[ u(x_t, 1) = x'_t \theta_1 \quad \text{and} \quad u(x_t, 0) = x'_t \theta_0, \]

These three assumptions—additive separability \( u(s_t, a_t) = u(x_t, a_t) + \epsilon_t(a_t) \), conditional independence, and extreme value distribution—and their implications are explained in Appendix A1.

Generalization to a case with more than two choices is straightforward.
then, using the difference in the choice-specific value functions $D(x_t)$ and the state transition probabilities $p(x_{t+1}|x_t, a_t)$, both of which can be directly obtained from the data, the utility parameters for a given discount factor $\beta$ can be identified through the following equation:

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D(x_t) - \sum_{s=t+1}^{T} \beta^{s-t} \hat{E}[\log(1 + e^{-D(x_s)})|x_{t} = x, a = 1] + \sum_{s=t+1}^{T} \beta^{s-t} \hat{E}[\log(1 + e^{D(x_s)})|x_{t} = x, a = 0] = \sum_{s=t}^{T} \beta^{s-t} \hat{E}[x_{0s}|x_{t} = x, a = 0] \theta_0.
$$

The derivation of equation (4) is provided in Appendix A2.\footnote{This study, as most studies in the literature, treats time preference as exogenous. Becker and Mulligan (1997) show a different model where time preference can change endogenously as a result of individuals’ investment. This possibility is not considered here.}

The identification of the discount factor is based on Magnac and Thesmar (2002), who show that if there are certain exclusive restrictions that shift the expected future utilities (through, say, the transitions of state variables) without entering individuals’ instantaneous utility functions, then the discount factor can be identified. At this point, it is worth emphasizing that equation (4) is key to understanding the importance of incorporating subjective expectations into models of individual decision making. The left-hand side, functions of individuals’ choices that can be directly observed in the data, is clearly compatible with various combinations of state transition probabilities ($\hat{E}[x_{0s}|x_{t}, a]$) and utility and time preferences ($\theta$ and $\beta$) on the right-hand side. Assuming rational expectations is equivalent to setting the state transition probabilities equal to those observed in the data, which leads to one set of estimates of utility and time preferences. This paper relaxes this assumption and shows the importance of using individuals’ subjective state transition probabilities to obtain different estimates of utility and time preferences.

II. Data

This study uses the data from the Health and Retirement Study (HRS). The HRS is a nationally representative biennial panel survey. The baseline interviews were conducted in 1992 (wave 1) with birth cohorts 1931 through 1941 and their spouses, if married. New birth cohorts have been added to the initial sample of 12,652 persons in 7,702 households, and the most recent available data are from year 2010 (wave 10).\footnote{Appendix A3 shows the steps to derive $\sum_{s=0}^{T} \beta^{s-t} \hat{E}[x_{0s}|x_{t}, a]$.}

\footnote{The survey history and design are described in more details in Juster and Suzman (1995). Data flow and other information are also available at http://hrsonline.isr.umich.edu (last accessed on March 16, 2013). This paper uses the RAND HRS data, a cleaned and user-friendly version of the HRS data, produced by the RAND Center for the Study of Aging (see www.rand.org/labor/aging/dataprod/hrs-dtad.html, last accessed on March 16, 2013, for more}
A. Subjective Longevity Expectations

When deciding whether to engage in risky health behaviors, people face the tradeoff between short-term benefits (the instantaneous pleasure) and long-term costs (worse health and higher mortality rates in the future). One determinant of this trade-off is individuals’ subjective longevity expectations, which influence individuals’ health behaviors in a complex way. They may directly affect the present values of all the future utilities following each choice through agents’ time preference. They are also state-dependent, so the optimal health choices of the forward-looking agents further depend on the dynamic interactions between preferences, expectations, and future choices and states transitions. Here, we use a dynamic discrete choice model to analyze the impact of subjective longevity expectations on agents’ health-related behaviors.

Subjective longevity expectations used in this paper are obtained from survey responses to the following questions: “What is the percent chance that you will live to be 75 or more?” and “What is the percent chance that you will live to be 85 or more?” The first question is asked only to those under age 65 at the time of interview, and the second question is asked only to those under age 75. Because these reported subjective expectations are expressed in probabilities, they are intrapersonally and interpersonally comparable and relatively easy to interpret.

The validity of the reported subjective longevity expectations might be in question in at least the following two cases: when the responses are 0 percent, 50 percent, or 100 percent; or when the reported probabilities of living to age 75 do not exceed those of living to age 85.

The first case might occur if respondents round their answers to the closest integers. The second case is usually considered as implying that the respondents have made some mistakes or they have misunderstood the questions. It might also be attributable to rounding if the responses to the two questions are the same. In Section IIIC, we describe a new approach to analyze subjective longevity expectations data. This approach explicitly models the underlying data-generating process for the true subjective longevity expectations, utilizes interval responses to connect those true subjective longevity expectations with the reported ones, and therefore allows us to better address the potential reporting and measurement errors in the survey data.

13 The questions in 1992 were slightly different from those in the following waves: “Using any number from 0 to 10 where 0 equals absolutely no chance and 10 equals absolutely certain, what do you think are the chances that you will live to be 75 (85) or more?” For consistency across all waves, we convert 1992 answers to probabilities and make sure that responses from all waves fall within [0, 1]. Also note that, beginning from the fifth wave in 2000, the target age in the second longevity question has been based on respondent’s age at the time of interview, instead of being fixed at 85 for everyone.


15 Online Appendix Table 1 shows the percentages of observations in the final analysis sample with any of the special responses.
B. Smoking Behaviors

From the first survey in 1992, the HRS has been asking respondents about their smoking behaviors using the questions: “Have you ever smoked cigarettes? [NOTE: By smoking we mean more than 100 cigarettes in R’S lifetime; do not include pipes or cigars.]” and “Do you smoke cigarettes now?” Based on the answers to these two questions, the respondents can be categorized into current, former, and never smokers.

Smoking initiation among adult never smokers is virtually zero, and most variation in aggregate demand for cigarettes is attributable to the decision of whether to quit smoking at age 50 and above (Sloan, Smith, and Taylor 2003), which is also the main age cohort in our dataset. We therefore exclude never smokers from the analysis and focus on the decisions of those current and former smokers to quit or continue smoking. This group of current and former mature smokers, or adults who have ever smoked, will be referred to as “adult smokers” hereafter.

C. Additional Explanatory Variables

In addition to subjective longevity expectations and smoking choices, we also collect information on individuals’ health, genetic makeup, and other demographic characteristics.

Self-reported health status is measured using a 5-point Likert scale. Respondents were asked: “What do you think is your current health status: 1. excellent; 2. very good; 3. good; 4. fair, 5. poor?” To alleviate the potential problem with measurement errors, and to ease the estimation procedure, we summarize the information on self-reported health status with a binary indicator that equals one if the respondent is in bad health (fair or poor) and zero otherwise (excellent, very good, or good).16

We also control for a number of demographic characteristics, including respondents’ age, gender (female or not), race (non-Hispanic white or not), real household income in 1992 dollars (calculated using the Consumer Price Index), and the longevity of the respondent’s same-gender parent summarized by a binary variable set to 1 if the parent is still alive at the interview time or died at an age greater than 70, and 0 otherwise. This last variable proxies for the respondents’ private (genetic) information regarding their expected longevity. Admittedly, it would help to know the actual cause of the parent’s death in order to distinguish accidental deaths from choice-related and/or gene-related ones. Unfortunately, the HRS does not provide this information.17

One important characteristic of cigarette consumption is the long latency period between the time of smoking initiation and the onset of adverse health shocks18 yet

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16 See Bound (1991) for a comparison of self-reported and objective measures of health.
17 As a robustness check, we use only mothers’ or fathers’ longevity information for all the respondents, mothers’ longevity information for males, and fathers’ longevity information for females. Various cutoff ages are also explored. Results are qualitatively robust.
18 As documented by Hodgson (1992), the cumulative probability of survival is the same for males who have never smoked and male smokers at age 35. At age 45 (65, 85), the corresponding survival ratio between these two groups is 1.02 (1.18, 2.11).
even smokers who quit at age 65 can expect large gains in life years (Taylor et al. 2002). Therefore, we focus on 51-to-61-year-old adult smokers’ decisions to quit smoking.

Table 1 provides summary statistics for the final analysis sample, which includes 25,431 person-waves after excluding those with missing information on any of the aforementioned variables. Approximately 38 percent of the respondents in the sample are current smokers; the other 62 percent have smoked before. About 53 percent of the sample is female, with an average age of 56.5. Around 82 percent of the respondents are non-Hispanic whites. Seventy-four percent of our respondents’ same-gender parents are still alive at the interview time, or died after age 70. In period one, the average self-rated health level is 2.67, which is between “very good” and “good” health. The average household income is about $53.7K. Around 2 percent of the respondents die within two years. For those who survive, the average self-rated health level changes to 2.65, with average household income of $55.5K. The average subjective probabilities of living to ages 75 and 85 are 63 percent and 43 percent, respectively.

### III. Empirical Specification and Model Estimation

#### A. Empirical Specifications

The empirical specification in this paper follows Grossman’s 1972 health production model. In each period, adult smokers decide whether to continue \((a = 1)\) or to quit smoking \((a = 0)\) after they observe all the state variables \((X\text{ and } \varepsilon)\).
Their utilities are assumed to depend directly on their health status and (the logarithm of) their household income, as well as their smoking decisions, so the observable part of their instantaneous utility functions can be specified as:

\[ u(X, a) = \theta_{a0} + \theta_{a1} \text{bad health}_t + \theta_{a2} \ln(\text{household income}_t), \]

where household income captures the consumption of the composite good. Since discrete choices only depend on relative utility levels, following Arcidiacono, Sieg, and Sloan (2007), we normalize \( \theta_{12} = 1 \) and \( \theta_{1j} = 0 \) for \( j \neq 2 \); and if deceased, the adult smoker has zero utility. Given this normalization, we can rewrite the difference in instantaneous utilities as:

\[ u(X, 0) - u(X, 1) = \alpha_0 + \alpha_1 \text{bad health}_t + \alpha_2 \ln(\text{household income}_t), \]

where \( \alpha_0 \), \( \alpha_1 \), and \( \alpha_2 \) have the interpretation as the differences between smoking and not smoking in the instantaneous benefits of the smoking choices (\( \alpha_0 \)), the instantaneous costs of having bad health (\( \alpha_1 \)), and the instantaneous benefits of household income (\( \alpha_2 \)), respectively.

Adult smokers are uncertain about their future. Specifically, they are uncertain about their future health and income following different smoking decisions, and need to rely on their subjective expectations when making current choices.

The health transition probabilities have two parts because adult smokers care first about their probabilities of staying alive and then, conditional upon being alive, they are also concerned with their overall health status, captured by their self-reported health (having bad health or not). So, these probabilities can be expressed as \( p(\text{alive}_{t+1}|X_t, a_t) \) and \( p(\text{bad health}_{t+1}|\text{alive}_t, X_t, a_t) \). Similarly, the (conditional) transition probabilities for household income can be expressed as \( p(\text{household income}_{t+1}|\text{alive}_t, X_t, a_t) \). Here includes not only adult smokers’ health status and household income at time \( t \), which are linked to various decisions that have been made up to the current period and can thus be considered as “endogenous” determinants of ones’ state transitions, but also their age, gender, race, and same-gender parents’ longevity, which reflect adult smokers’ natural and biological initial conditions—the relatively “exogenous” factors. The last variable, as noted in Section II, is used to control for the differences in adult smokers’ expected longevity attributable to (unobserved) familial and genetic factors. This variable also serves as the exclusive restriction for the identification of the discount factor (see Section I). Specifically, we assume that same-gender parents’ longevity affects adult smokers’ own expectations about future survival, health, and income, but its impact on their instantaneous utilities does not differ by their smoking choices.

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20 Household income per capita is used instead as a part of the sensitivity test. Results are robust.
21 See Ehrlich and Yin (2005) for the economic rationale behind the variables included in this specification.
B. Estimation of State Transitions

In order to estimate utility and time preferences using the dynamic discrete choice model, as discussed in Section I, we need to first estimate one-period ahead subjective survival probabilities, conditional health transition probabilities, conditional income transition probabilities, and conditional choice probabilities. For comparison purposes, and to highlight the importance of incorporating subjective expectations data, we also discuss the estimation of these probabilities in an objective model based on the rational expectations assumption.

Under the rational expectations assumption, estimation of survival expectations is straightforward, because what is observed in the data is assumed to be what individuals expect. Therefore, for this objective estimation of survival expectations, we use a Logit model with observed survival status between two periods as the binary dependent variable, and control for the complete set of the observed state variables ($X$).

The more interesting part, naturally, is the estimation of the one-period ahead subjective survival probabilities without the rational expectations assumption, to which we devote the following subsection.

Ideally, investigating the effects of subjective expectations on individuals’ decision-making process would require information on individuals’ assessments of all the possible outcomes with all the possible combinations of state and choice variables. That is, the ideal estimation requires subjective evaluations of all the components we use in the dynamic discrete choice model, whether it is conditional health and income transition probabilities or unconditional one-period ahead survival probabilities. However, it is currently impractical to gather such complex information from survey respondents. Therefore, some parts of the estimation for subjective and objective models are necessarily the same and based on observed data. Specifically, conditional health and income transition probabilities are estimated nonparametrically using the complete set of observed state variables, and conditional choice probabilities are estimated flexibly using a Logit model, with up to fourth-order polynomials and interaction terms of the observed state variables.

C. Estimation of Subjective Survival Probabilities

We are faced with two issues when estimating one-period (i.e., two-years) ahead subjective survival probabilities. First, as mentioned in Section II, the reported subjective longevity expectations might be subject to potential measurement and reporting errors in certain cases, and this issue needs to be addressed in the estimation. Second, our respondents were asked to report subjective probabilities of surviving to the target ages (75 and 85) at least 10 years in the future. However, for these reported subjective expectations to be directly used in the dynamic discrete choice model, they need to be “translated” into the corresponding subjective probabilities of surviving another two years in the future.

Let us start with the first issue. Assume for now that we actually have the reported subjective probabilities of surviving another two years for respondent $i$, denoted as
$p_i(2)$, which can be different from the respondent’s true subjective two-year survival probabilities $p_i'(2)$ by an additive noise $\nu_i$:

$$p_i(2) = p_i'(2) + \nu_i,$$

where $\nu_i$ reflects the respondent’s reporting or rounding error. To identify the true subjective two-year survival probabilities in the data, we further assume that they follow a certain data generating process:

$$p_i'(2) = g(x_i \delta),$$

where $g(x_i \delta)$ is the Logistic function,

$$g(x_i \delta) = \exp(x_i \delta)/(1 + \exp(x_i \delta)),$$

with $x_i$ and $\delta$ being the vectors of observable state variables and model parameters, respectively, and $0 \leq g(x_i, \delta) \leq 1$. The parameter vector $\delta$, measuring the effect of the observables on two-year survival probabilities, is the key input for the dynamic discrete choice model. To estimate $\delta$ using information available in our survey data, we employ an approach similar to that in Manski and Molinari (2010) and Hudomiet, Kezdi, and Willis (2011), and consider interval responses instead of point probabilities to capture the reported expectations.

Specifically, each of the reported two-year subjective survival probability for respondent $i$ ($p_i(2)$) is assumed to be in a prespecified interval $[L_{i,2}, U_{i,2}]$, where $L_{i,2}$ and $U_{i,2}$ refer to the lower and upper bounds of this two-year response interval for agent $i$, and this agent’s true subjective survival probability is assumed to be in the same interval but not necessarily equal to the reported one due to the noise $\nu_i$. That is, we do not rely on the actual value of the reported point probability, which can be subject to measurement or reporting errors, but instead use the corresponding response interval to identify $\delta$.

To obtain point estimates of $\delta$, as in Hudomiet, Kezdi, and Willis (2011), we assume that the noise $\nu_i$ has a Normal distribution with mean zero and variance $\sigma^2$. Under this assumption, the likelihood of observing the reported probability $p_i(2)$ given $x_i$ is:

$$L_i(x_i, \delta) = \Pr(L_{i,2} \leq p_{i,2} \leq U_{i,2})$$

$$= \Pr(L_{i,2} - g(x_i \delta) \leq \nu_i \leq U_{i,2} - g(x_i \delta))$$

$$= \Phi\left(\frac{U_{i,2} - g(x_i \delta)}{\sigma}\right) - \Phi\left(\frac{L_{i,2} - g(x_i \delta)}{\sigma}\right),$$

where $L_i$ is the likelihood for respondent $i$ and $\Phi$ is the CDF of a standard Normal distribution. The likelihood function for all the individuals then satisfies

$$L(x, \delta) = \prod_{i=1}^{N} L_i,$$
which is maximized to obtain the point estimate of $\delta$.

After dealing with the first issue by explicitly modeling the underlying data generating process for the true subjective survival probabilities and then connecting them to the reported ones using interval responses, we can now address the discrepancy in time horizon between the reported subjective survival expectations (at least ten years in the future) and the ones needed in our model (two years in the future). Following the literature (e.g., Khwaja, Sloan, and Chung 2007 and Perozek 2008), in our benchmark analysis, we use a subjective hazard function with a Weibull distribution to model respondents’ survival curves and infer their two-year survival probabilities.22

Specifically, respondent $i$’s subjective expectation for surviving another $t$ years, $p_i(t)$, is specified as

$$p_i(t) = \exp(-\gamma_i t^k_i),$$

where $k_i$ and $\gamma_i$, the shape and scale parameters, can be estimated using the two reported subjective probabilities of living to ages 75 and 85. That is, for each respondent, we have

$$p_i(t_{i,75}) = \exp(-\gamma_i t_{i,75}^k_i),$$

$$p_i(t_{i,85}) = \exp(-\gamma_i t_{i,85}^k_i),$$

where $t_{i,75}$ and $t_{i,85}$ are years from the time of interview to ages 75 and 85 for individual $i$, respectively. This system of equations is then used to solve for $k_i$ and $\gamma_i$23 which are subsequently used to infer the subjective two-year survival expectation for respondent $i$:

$$p_i(2) = \exp(-\gamma_i 2^k_i).$$

Next, we construct the intervals for the reported subjective expectations of surviving to ages 75 and 85, $[L_{i,t_{i,75}}, U_{i,t_{i,75}}]$ and $[L_{i,t_{i,85}}, U_{i,t_{i,85}}]$, to obtain the interval for the two-year survival probability, $[L_{i,2}, U_{i,2}].$24 That is, we allow the true survival probabilities to lie anywhere inside the intervals, and then solve for the corresponding scale and shape parameters of the Weibull hazard distribution to obtain the

---

22 Weibull distribution imposes a monotonicity property on the mortality hazard, and US National Center for Health Statistics data show that mortality hazard in the US is indeed monotonically increasing. See, for example, http://www.cdc.gov/nchs/data/nvsr/nvsr48/nvsr48_18.pdf (last accessed on March 16, 2013).

23 Specifically, for each respondent $i$, we can solve for $k_i$ through

$$\frac{\ln[p_i(t_{i,75})]}{\ln[p_i(t_{i,85})]} = \frac{t_{i,75}^k}{t_{i,85}^k} \Rightarrow \ln[p_i(t_{i,75})] = k_i \ln[t_{i,75}] - k_i \ln[t_{i,85}].$$

After we recover $k_i$ for each individual, we can plug it back into the expression for $p_i(t_{i,75})$ (or $p_i(t_{i,85})$) and solve for $\gamma_i$.

24 Because a large fraction of the responses are multiples of 5 and 10 percent, we define the following 10 percentage point wide intervals for the probability of survival to ages 75 and 85: $[0, 5); [5, 15); ... [95, 100].$
implied subjective probability of surviving another two years\(^{25}\). The minimum and maximum of these implied probabilities then become the lower and upper bounds for the two-year intervals \([L_{i,2}, U_{i,2}]\) used in the estimation of \(\delta\) in equation (5).

Our method of dealing with the subjective expectations data, as described above, makes it perfectly reasonable to have 0 percent, 50 percent, 100 percent, or any multiples of 5 percent or 10 percent as responses, instances seemingly questionable at the first glance. Even the cases where \(p_i(t_i,75) \leq p_i(t_i,85)\) may be rationalized if \(p'_i(t_i,75) > p'_i(t_i,85)\), but both true probabilities fall in the same interval and the noise for \(p'_i(t_i,75)\) is smaller than that for \(p'_i(t_i,85)\). Our method also solves the discrepancy in time horizon between the reported subjective expectations and those required in the estimation of the dynamic discrete choice model. Furthermore, because our model is fully specified, we can obtain point estimates for the parameters of interest instead of potentially wide sets of parameter estimates, which could be derived using a method of partial identification, as discussed in Manski and Molinari (2010). Finally, our method makes it possible to employ subjective expectations data directly in a dynamic discrete choice model, and therefore complements the method used by Van der Klaauw and Wolpin (2008) who take subjective longevity expectations as auxiliary data to augment the limited information on observed states and choices.

In Section IIIE, we show how this method can be extended to capture persistent and unobservable heterogeneity in individual expectations and preferences, and in Section IV we discuss the empirical findings and their robustness to various model specifications.

D. Estimation of Utility Parameters and Discount Factor

After dealing with conditional choice probabilities, conditional state transitions, and survival probabilities, we now move on to the estimation of the utility parameters and the discount factor, using a method standard in the literature (see, for example, Hotz and Miller 1993). Specifically, with the normalization specified in Section IIIA, for each possible value of the discount factor \(\beta \in [0,1]\), we obtain the utility parameter estimates \(\hat{\alpha}\) using equation (4) by regressing the left-hand side dependent variable, which only contains information directly obtainable from the data, on the right-hand side generated regressors. Given the utility parameter estimates at each value of \(\beta (\in [0,1])\), we then use a linear line search to locate the optimal discount factor with the maximum likelihood.

So, together with the preceding subsections, the overall estimation process consists of the following steps:

Step 1: Estimate survival probabilities, conditional health and income transition probabilities, and conditional choice probabilities, as discussed in Sections IIIB and IIIC.

\(^{25}\)In practice, we discretize these two intervals into 10 cells and match the midpoint of each cell for \(p_i(t_i,75)\) with the midpoints from all the cells for \(p_i(t_i,85)\) to obtain 100 sets of estimated scale and shape parameters. We choose the set with the largest interval for subjective two-year survival probabilities.
Step 2: For each discount factor $\beta \in [0, 1]$, recover $\hat{\alpha}$ in equation (4) (after normalization) using estimates from Step 1.

Step 3: Construct the sample likelihood at the discount factor $\beta$ specified in Step 2 and its corresponding $\hat{\alpha}(\beta)$.

Step 4: Repeat Steps 2 and 3 for each discount factor $\beta$ along a line search to locate the $\beta$, and the corresponding utility parameter estimates, with the maximum likelihood.

E. Unobserved Heterogeneity

The estimation procedure discussed in the preceding subsections implicitly assumes away any persistent unobserved heterogeneity in state transitions and utility and time preferences, which might lead to inconsistent estimators of model parameters. This estimation procedure also relies on the potentially strong assumption that those publicly observed variables we control for in our model can fully capture all the relevant states in individuals’ dynamic decision-making processes, which, if violated, would cause endogeneity bias. We therefore introduce to our model persistent unobserved heterogeneity in state transitions and preference parameters to deal with the potential endogeneity issue, following the insights from Keane and Wolpin (1997) and Kasahara and Shimotsu (2009), and using an empirical method similar to that by Arcidiacono, Sieg, and Sloan (2007).

Specifically, we allow individuals to belong in different types with persistent differences, which are not captured by the publicly observable states, and identify these types using subjective survival expectations data. We assume there are $J$ distinct types whose subjective survival probabilities exhibit different sensitivities to the observables, so the likelihood function for the reported subjective probabilities for individual $i$ of type $j$ can be denoted as $L_{ij}(x_i, \delta_j)$, which is defined similarly to equation (6) but allows the parameters to be type-specific. The total likelihood function for individual $i$ integrates out the $J$ different types:

$$L_i(x_i, \{\delta_j\}_{j=1}^J) = \prod_{j=1}^J \pi_j L_{ij}(x_i, \delta_j),$$

where $\pi_j (\in [0, 1])$ denotes the relative share of type $j$, with $\sum_{j=1}^J \pi_j = 1$, and is assumed to be constant for simplicity. Then, the type-specific parameters and the shares are estimated by maximizing the likelihood function over the whole sample,

$$L(x, \{\delta_j\}_{j=1}^J) = \prod_{i=1}^N L_i(x_i, \{\delta_j\}_{j=1}^J).$$

Once we have $\hat{\delta}_j$, we can assign each individual to the type with the highest likelihood, and then estimate the parameters in conditional state transitions and utility and time preferences within each type separately, following the approach discussed in Section IIID.
Estimating the model sequentially allows us to identify the unobserved types and the model parameters in a tractable and easy-to-interpret way. Because the identification of these types in our model is based on the subjective expectations data, we can capture the persistent heterogeneity in agent’s longevity expectations, which cannot be explained by their observed states, by interpreting these types as group fixed effects so that individuals within each type share similar dynamics of longevity expectations. Furthermore, as the time-invariant factors, which affect subjective expectations, are also likely to impact individual choices, carrying out the estimation of model parameters within each type separately also helps alleviate the potential endogeneity issue caused by the possibility that the set of observable variables in our model might not fully capture the formation of subjective survival expectations.

IV. Results

A. State Transition Probabilities \( (p(s_{t+1}|s_t, a_t)) \)

We estimate nonparametrically the income and health transition probabilities conditional upon survival. Figure 1 shows the estimated conditional two-year health transitions as a function of (the logarithm of) household income for different combinations of health status, parents’ longevity, and smoking choices. As expected, individuals who are currently in bad health or do not have long-lived parents are more likely to have bad health in the next period (Figure 1, panel A and panel B). Smoking today also increases
one’s chance of having bad health tomorrow, regardless of the current health status (Figure 1, panel C and panel D). As is clear from all the panels in Figure 1 and consistent with the existing literature on income-health gradients, higher household incomes now predict a lower probability of having bad health in the future.26

Table 2 reports the estimated probabilities of surviving to the next period. The first two columns are objective estimates, based on the rational expectations assumption, with observed survival between two periods as the dependent variable. The results are as expected: smoking, bad health, and aging reduce one’s probability of surviving another two years, and white females with higher household incomes and long-lived parents are more likely to stay alive in two years. All of the estimates are statistically significant at the 1 percent level, except for same-gender parents’ longevity and being non-Hispanic white. That is, according to the observed mortality data and after controlling for other individual characteristics and behaviors, one’s race and same-gender parents’ longevity are not statistically significant determinants of survival.

The last two columns of Table 2 present the subjective estimates of the probabilities of surviving another two years, based on the method discussed in Section IIIC. Note that, except for “being female,” every estimate has the same sign as its objective counterpart. That is, people’s subjective survival expectations are generally consistent with observed mortality data. Some other studies, using different methods in different contexts, also find that individuals’ expectations about the effects of different determinants on survival are generally similar to those observed in the data (see, for example, Hurd, McFadden, and Merrill (2001); Smith, Taylor, and Sloan (2001); and Hurd and McGarry (2002)).

At least two main differences between the objective and the subjective estimates of two-year survival probabilities are noteworthy. The first one lies in the statistical significance of same-gender parent’s longevity and being white. Objectively, both determinants are positive but neither is statistically significant; subjectively,
however, they are highly significant. The second main difference between these two sets of parameter estimates is in their relative magnitude. For example, objectively, current health condition matters the most for one’s survival in two years, having a long-lived parent can only “cancel out” less than half of the negative effect of smoking (in a statistically insignificant way), and being a smoker now is equivalent to being at least four years older in terms of its negative effects on two-year survival. Subjectively, however, having bad health is no longer the largest threat to one’s survival; it is actually almost equivalent in magnitude to having a long-lived parent, which in turn can “cancel out” more than half of the detrimental effect of smoking. And smoking, subjectively, is only comparable to ageing for about two years.

Regardless of the sources of these differences, whether they are attributable to valuable private information unobservable to economists or actually reflect individuals’ systematic mistakes, it is clear that individuals form longevity expectations differently than described by the rational expectations assumption, and ignoring these differences could result in inconsistent estimators of utility and time preferences, as we will show momentarily.

To illustrate the similarities and, more importantly, the differences between the subjective and the objective survival expectations, Figure 2 presents the two-year survival probabilities for white males with long-lived parents. The right two panels are based on subjective survival probabilities estimated using the method discussed in Section IIIC, and the left two panels use objective estimates from the rational expectations model. We can see that both subjective and objective estimates suggest that survival probabilities increase with household income and are lower for current smokers. However, for those who are currently in bad health (panels A and B), subjective survival probabilities are everywhere higher than the objective ones, and the difference in two-year survival probabilities by smoking status is smaller for the subjective estimates than for the objective ones. For those who are currently in good health (panels C and D), the pattern is reversed: subjective survival probabilities are everywhere lower than the objective ones, and the difference in two-year survival probabilities by smoking status is greater for the subjective estimates than for the objective ones. The emphasis here again is on the existence of, not the reasons behind, these differences between subjective and objective survival probabilities which, if ignored, would lead to inconsistent estimators of utility and time preferences.

B. Utility and Time Preferences \((\alpha \text{ and } \beta)\)

Table 3 reports the objective and the subjective estimates of utility and time preferences and the differences between them. We can observe the following similarities

---

27 There is actually mixed evidence in the literature on the predicting power of parents’ longevity for one’s own longevity. Many studies show that longevity might be hereditary (e.g., Frederiksen et al. 2002 and Gjonca and Zaninotto 2008), while some others find no evidence of this kind of predicting power (e.g., Friedman and Martin 2011).

28 Bootstrapped standard errors are also reported. Given the relatively complex nature of our estimation (the first step estimates two-year subjective survival probabilities using interval responses and a hazard model, followed by the second-step estimation of utility parameters and time preference), the bootstrap approach to derive the standard errors seems the most appropriate (see, for example, Chaudhuri, Goldberg, and Jia 2006). The potential disadvantage of this approach is a loss in efficiency, but the bootstrapped standard errors turn out to be reasonable and similar to (but somewhat larger in magnitude than) the asymptotic standard errors, ignoring the first-step estimation. And
as is apparent in Table 3, the model parameters are fairly precisely estimated. We use 200 bootstrapped samples because empirical evidence suggests that 200 replications are sufficient in most cases to estimate the standard errors (Efron and Tibshirani 1994).

Table 3—Utility Parameters and Discount Factor

<table>
<thead>
<tr>
<th>Variables</th>
<th>Objective estimates</th>
<th>Subjective estimates</th>
<th>Difference in estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Standard error</td>
<td>Coefficient</td>
</tr>
<tr>
<td>Bad health</td>
<td>−1.031***</td>
<td>0.150</td>
<td>−0.404***</td>
</tr>
<tr>
<td>log(household income)</td>
<td>1.486***</td>
<td>0.026</td>
<td>1.531***</td>
</tr>
<tr>
<td>Constant</td>
<td>−5.301***</td>
<td>0.262</td>
<td>−6.107***</td>
</tr>
<tr>
<td>β</td>
<td>0.860***</td>
<td>0.023</td>
<td>0.980***</td>
</tr>
<tr>
<td>Observation</td>
<td>25,431</td>
<td></td>
<td>25,431</td>
</tr>
</tbody>
</table>

Notes: Objective estimation is based on the rational expectations assumption. Subjective estimation uses subjective longevity expectations. Reported standard errors are based on 200 bootstrapped samples.*** Significant at the 1 percent level.

Notes: Probabilities of surviving another two years as a function of the household income by smoking status for white males. Left panels: objective estimation using observed mortality data. Right panels: subjective estimation using reported subjective expectations data.
and differences between these two sets of estimates. The first similarity lies in the negative sign of having bad health, indicating that subjectively and objectively bad health is associated with a larger instantaneous utility loss if one chooses to quit smoking, a finding consistent with the existing literature (e.g., Khwaja, Sloan, and Wang 2009). Given the adverse effects of smoking on one’s health, for smoking to be desirable, the disutility from bad health when smoking should be less than that when not smoking. In other words, health status is given less attention when one decides to continue smoking.

Second, both subjective and objective estimates of the instantaneous utility of household income are positive and greater than one, implying that: (i) the marginal utility of income is higher if one chooses to quit smoking than if one chooses to continue smoking; and (ii) the difference in individuals’ instantaneous utilities by smoking status increases with household income, so that people with higher income will experience a greater instantaneous utility loss if they choose to continue smoking than will their lower-income counterparts. These findings concur with previous studies showing that smoking tends to be more prevalent among low-income populations (e.g., Chaloupka and Warner 2000).

When turning our attention to the differences between the subjective and the objective estimates, we see that these two sets are statistically significantly different at the 1 percent level (Table 3, last two columns), except for household income. Two main observations are particularly interesting. First, although sharing the same negative sign as the subjective one, the objective estimate of having bad health is more than twice as large, meaning the difference in the instantaneous utility loss associated with having bad health between the two choices (of quitting or continuing smoking) is much smaller in subjective case than under the rational expectations assumption. That is, to rationalize smoking choices under rational expectations, smoking is found to be associated with a much smaller disutility than nonsmoking when in bad health. On the other hand, under subjective expectations, while the disutility from bad health is still lower when smoking compared to not smoking, the gap in utility sensitivity to health between these two smoking choices is substantially reduced.

Second, the discount factors are estimated to be 0.86 under the rational expectations assumption and 0.98 using the subjective expectations data. Both are within the reasonable range suggested by previous studies (e.g., Moore and Viscusi 1988 and Arcidiacono, Sieg, and Sloan 2007), evidence that adult smokers in our sample are indeed forward-looking. However, the discount factor estimated using the subjective expectations data is significantly larger than the objective one, indicating that individuals are actually much more forward-looking, or much more patient with time, than suggested by a rational expectations framework. Without the subjective expectations data, we would have underestimated adult smokers’ time preference.

Combining these estimation results on utility and time preferences with those on survival probabilities from the preceding subsection, we reach at least three important conclusions about our sample of adult smokers. First, even when deciding to smoke, they care greatly about their health, much more than we would have concluded without using subjective expectations data. Second, they are much more forward-looking than they would appear in a model assuming rational expectations. And third, the reason that a rational expectations framework would have underestimated
adult smokers’ preference for health and time is that this framework ignores the differences between subjective and objective survival expectations.

**C. Unobserved Heterogeneity**

As discussed in Section IIIE, we introduce heterogeneity to our model to allow for persistent unobserved differences in state transitions and utility and time preferences and to deal with the potential endogeneity issue. Specifically, we estimate our model with two distinct types using the method discussed in Section IIIE, and our estimation results show that incorporating unobserved heterogeneity can indeed improve our understanding of individuals’ expectations and preferences.

Table 4 reports the summary statistics for these two types. The Type I subsample, consisting of 82 percent of the whole sample, is more likely than the Type II subsample to be a young healthy white female with a high family income and a long-lived same-gender parent. People belonging in the first type are less likely to be smoking at the time of the interview than those in the second type (0.37 versus 0.40); they also have much higher subjective expectations of living to ages 75 (0.68 versus 0.31) and 85 (0.45 versus 0.29).

Table 5 shows the means and the standard deviations, by smoking status, of the reported subjective expectations of living to age 75 for the whole sample and by type. Clearly, the standard deviations for each type, whether by smoking status...
or not, are smaller than those for the whole sample, indicating that individuals within each type are indeed more similar to each other in their subjective longevity expectations than to people of the other type, a pattern consistent with our expectation and our estimation method.

Panel A of Table 6 shows the estimated two-year subjective survival probabilities for these two types. Overall, these two types respond in the same manner to all the determinants of two-year survival. However, the magnitudes of these effects can be quite different across the types. As we can see, compared to Type I, Type II attaches more weight to relatively “exogenous” determinants of survival, such as parents’ longevity, race, and age, and less weight to relatively “endogenous” factors, such as smoking, health, and income. For example, Type I believes that smoking has the largest negative effect on their survival, with its magnitude three times as large as that of parent’s longevity, while Type II attaches larger weight to parent’s longevity than to smoking.

Panel B of Table 6 shows the estimates of the utility and time preferences for these two types. Compared to Type II, Type I is found to associate higher instantaneous utility loss with bad health (|−0.37| < |−1.75|) and higher income (1.51 > 1.19 > 1) if they choose to smoke, enjoy higher instantaneous benefits from quitting smoking (|−5.64| > |−1.52|), and be more forward-looking (0.91 > 0.88). Overall, these

In addition, when we regress the reported subjective expectations of living to age 75 on all the observed state variables and next period’s smoking choices using a sample of individuals who survive to the second period (because only they can make smoking decisions the next period), we see that for the overall sample without persistent unobserved heterogeneity, the following period’s smoking status is a statistically significant predictor of current subjective survival probabilities, evidence of the correlation between current subjective expectations and future smoking choices. However, after we allow for two different types, within each type, the following period’s smoking status is no longer a statistically significant predictor of current subjective probabilities of survival. That is, introducing unobserved heterogeneity can indeed help alleviate the potential endogeneity issue. Results are available upon request.
findings suggest that private information in agents’ subjective expectations not captured by publicly observed information plays an important role in our understanding of their preferences and behaviors.

D. Goodness of Fit

Figure 3 presents two measures of within-sample goodness of fit: the model predicted smoking rates versus those observed in the data by income (left panel) and by age (right panel). Overall, these two panels show that both estimation methods, the one using subjective expectations data with unobserved heterogeneity and the one based on the rational expectations assumption, are able to fit the data well. Both sets of predicted smoking rates are largely within the 95 percent confidence interval around the data.30

30 Measures of within-sample goodness of fit based on subjective expectations data without unobserved heterogeneity are qualitatively similar and available upon request. Online Appendix Table 5 provides more information on the comparison between subjective (with unobserved heterogeneity) and objective models in their abilities to fit the data, and the conclusion holds that both models perform quite well.
The finding that the subjective estimates fit the data very well, even without assuming rational expectations, concurs with a few other studies that use subjective expectations data. Gan et al. (2004), using data from the Asset and Health Dynamics Among the Oldest Old, also find that parameter estimates using subjective mortality risks perform better in predicting out-of-sample wealth levels than estimates based on life table mortality risks. Similarly, Lochner (2007) shows that perceived crime rates can predict a youth’s criminal behavior better than the official neighborhood crime rates. These findings are important because they imply that goodness of fit should not be a concern when choosing between rational expectations assumption and subjective expectations data. Goodness of fit alone cannot justify the choice of the former over the latter.

E. Counterfactual Experiment

Utility and time preferences estimated using subjective longevity expectations can be used to simulate the impact of various counterfactual experiments on smoking choices. For example, what would happen to the smoking rates if the rational expectations assumption is true, that is, subjective survival expectations are indeed the same as those estimated using an objective model? To answer this question, we conduct an experiment where people in our sample have the utility and time preferences recovered using the subjective estimation with unobserved heterogeneity, but hold the objective survival expectations estimated using the rational expectations framework. Figure 4 shows the result of this counterfactual experiment, with smoking rates by income on the left and by age on the right. Both panels clearly show that, when adult smokers’ subjective longevity expectations are set to be equal to the objective ones, smoking rates are lower than those observed in the data, 8 percentage points lower, on average. This difference in smoking rates also varies by income and age. Adult smokers with lower household incomes experience a larger difference than their higher income counterparts (left panel), and this difference increases slightly with age (right panel).

Effects of this counterfactual experiment on smoking rates by different subgroups are also shown in online Appendix Table 6. For all subgroups, the counterfactual experiment predicts a lower smoking rate, ranging from about 18 percent to 31 percent of the original rate, with nonwhite, male, those with bad health, and those whose parents died early seeing a greater change in smoking rates than their counterparts.

These results can be interpreted in two ways, depending on how we understand the difference between subjective and objective longevity expectations. If we believe that subjective expectations capture valuable private information, which is relevant in individuals’ decision-making process but unobservable to economists, then the smoking rates predicted by the experiment (30 percent on average) are the part of the observed smoking (38 percent, see Table 1) that can be explained by a model that has the true (i.e., subjectively estimated) utility and time preferences and assumes rational expectations, while the difference (38 − 30 = 8 percent) between the predicted smoking rates and the ones actually observed in the data is the part of the observed smoking behavior such a model cannot explain.

On the other hand, if we believe individuals have made systematic mistakes in their subjective expectations, then the reduction in smoking rates predicted by the
experiment shows what can be achieved by public policies aimed at further reducing the smoking rates, such as an information campaign that matches individuals’ subjective survival expectations with the objective ones.\footnote{Various policies have been proposed and implemented to reduce smoking. The most studied one is cigarette taxation. Chaloupka and Warner (2000) and Cawley and Ruhm (2011) summarize the rationale for cigarette excise taxes with respect to their revenue potential and negative impact on consumption. Other policies include federal and state counter-advertising campaigns and restrictions on tobacco advertising (e.g., Hu, Sung, and Keeler 1995 and Emery et al. 2007), smoking ban at workplaces and restaurants (e.g., Wasserman et al. 1991 and Carpenter 2009), and comprehensive smoke-free air laws (e.g., Adda and Cornaglia 2010).} Actually, some simple back-of-the-envelope calculations show that this reduction in smoking rates could mean a gain in life years worth $3.07 trillion.\footnote{In 2010, the US population was roughly 309 million (http://factfinder2.census.gov/faces/tableservices/jsf/pages/productview.xhtml?pid=DEC_10_DP_DPDP1, last accessed on 03/16/2013). Ten percent of the population was between 50 and 60 years of age, and according to the HRS data, about 62 percent of them had smoked at some point in their lives. Men can gain an average of 1.4 to 2.0 years of life, and women 2.7 to 3.7 years, when they quit smoking at age 65 (Taylor et al. 2002). These gains are greater if smokers quit at a younger age. To be conservative, let us use 2 years as the average gain from quitting smoking for our age group (51–61) and $100,000 as the value for one quality-adjusted life year (Cutler 2004). Because an additional 8 percent or so of this age group would}
costs of externalities associated with cigarette consumption, such as air pollution or second-hand smoking (Sloan et al. 2004).

F. Robustness Checks

In this section, we discuss the results of several robustness checks. First, as discussed in Section IIIC, we can identify the shape and scale parameters in the hazard function using respondents’ subjective expectations of living to ages 75 and 85. However, as pointed out by Perozek (2008), when these two expectations reported by the same individual are sufficiently close, the exactly identified hazard functions might be implausibly flat, implying unreasonably high probabilities of surviving to old ages. Therefore, as a robustness check, we impose the additional assumption that the probability of surviving to age 110 is essentially 0, so now the Weibull hazard distribution for each individual is over-identified with two unknowns and three equations.

The estimation results for this first robustness check are shown in Table 7, and they are qualitatively similar to those from the main specification. We can still conclude that individuals’ utility and time preferences are different from those estimated in a rational expectations framework, and these differences are due to the discrepancy in longevity expectations between subjective and objective estimations.

The second robustness check looks into the hazard function itself. We use a Weibull hazard function when estimating the two-year subjective survival probabilities in our main specification (Section IIIC). To check whether our results are robust to other hazard functions, here we replace the Weibull hazard function with another

<table>
<thead>
<tr>
<th>Variables</th>
<th>Type I</th>
<th>Type II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Standard error</td>
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<tr>
<td><strong>Panel A. Two-year survival probabilities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Currently smoke</td>
<td>−0.263***</td>
<td>0.019</td>
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<tr>
<td>Same-gender parent’s longevity</td>
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<tr>
<td>Bad health</td>
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<td>log(household income)</td>
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<td>Non-Hispanic white</td>
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<td>Female</td>
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<td>Age</td>
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<td><strong>Panel B. Utility and time preferences</strong></td>
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<tr>
<td>Bad health</td>
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<td>(\beta)</td>
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<td>0.010</td>
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<tr>
<td>Share (percent)</td>
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<td>8</td>
</tr>
</tbody>
</table>

Note: Estimation results reported in this table are based on the method described in Section IIIE. ***Significant at the 1 percent level.

quit smoking according to this experiment, we will gain about 309 × 10 percent × 62 percent × 8 percent × 2 \(= 3.07\) million life years, which are worth \(3.07 \times \$100,000 = \$3.07\) trillion.
hazard function commonly used in the literature—the Gompertz hazard function (e.g., Perozek 2008). The estimation procedure is the same as that for the Weibull hazard function, and the estimates for the subjective survival expectations and utility and time preferences with unobserved heterogeneity are reported in Table 8. Again, we see the same pattern in the subjective longevity expectations and the utility and time preferences as that from the main specification.

The third and fourth robustness checks touch upon two specification choices we have made while estimating the two-year subjective longevity expectations. First, we have used 10 percentage point wide intervals ($[0, 5]; [5, 15]; \ldots; [95, 100]$) when connecting the true underlying subjective expectations with the reported ones (see footnote 24). To check whether our results are sensitive to this assumption, here we try 15 percentage point wide intervals ($[0, 5]; [5, 20]; [20, 35]; \ldots; [95, 100]$) instead. Second, as explained in footnote 25, for each individual, we discretize the two intervals for subjective expectations of living to ages 75 and 85 into 10 cells and then match them to obtain scale and shape parameters for the subjective hazard function. Here, we use only five cells and check whether results are qualitatively different. Results for these two robustness checks are reported in online Appendix Tables 7 and 8, respectively, and they are indeed qualitatively similar to the results from the main specification.

\[ M_i(t_i) = \exp[\gamma_i/k_i(1 - \exp(k_i t_i))], \]

where $k_i$ and $\gamma_i$ are the shape and scale parameters.

### Table 8—Robustness Check: Gompertz Hazard Function

<table>
<thead>
<tr>
<th>Variables</th>
<th>Type I</th>
<th>Type II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Standard error</td>
</tr>
<tr>
<td><strong>Panel A. Two-year survival probabilities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Currently smoke</td>
<td>$-0.200^{***}$</td>
<td>0.030</td>
</tr>
<tr>
<td>Same-gender parent’s longevity</td>
<td>$0.173^{***}$</td>
<td>0.033</td>
</tr>
<tr>
<td>Bad health</td>
<td>$-0.603^{***}$</td>
<td>0.049</td>
</tr>
<tr>
<td>log(household income)</td>
<td>$0.042^{***}$</td>
<td>0.016</td>
</tr>
<tr>
<td>Non-Hispanic white</td>
<td>$-0.004$</td>
<td>0.033</td>
</tr>
<tr>
<td>Female</td>
<td>$0.092^{***}$</td>
<td>0.027</td>
</tr>
<tr>
<td>Age</td>
<td>$-0.006^{***}$</td>
<td>0.008</td>
</tr>
<tr>
<td>Constant</td>
<td>$7.568^{***}$</td>
<td>0.387</td>
</tr>
<tr>
<td><strong>Panel B: Utility and Time Preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bad health</td>
<td>$-0.526^{***}$</td>
<td>0.040</td>
</tr>
<tr>
<td>log(household income)</td>
<td>$1.418^{***}$</td>
<td>0.021</td>
</tr>
<tr>
<td>Constant</td>
<td>$-4.542^{***}$</td>
<td>0.335</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$0.940^{***}$</td>
<td>0.021</td>
</tr>
</tbody>
</table>

**Note:** Estimation results reported in this table are based on the method described in Section IIIE. ***Significant at the 1 percent level.
The last robustness check deals with some seemingly unreasonable subjective longevity expectations reported by our respondents. As discussed in Section IIIC, our method of estimating two-year survival probabilities can rationalize certain special cases of reported subjective expectations that could otherwise be considered as mistakes or misunderstandings of the respondents. Here, we estimate the model again without those respondents whose reported subjective expectations of living to ages 75 and 85 are both equal to 0, 50, or 100 percent. The estimates of the subjective survival probabilities and utility and time preferences with unobserved heterogeneity are reported in online Appendix Table 9. All the results are qualitatively similar to those using the actual final analysis sample.

V. Conclusion and Discussion

In this paper, we demonstrate how to relax the rational expectations assumption in a dynamic discrete choice model by incorporating subjective expectations data. We also propose a new method to deal with these subjective expectations data, and explain how to allow for persistent unobserved heterogeneity in state transitions and utility and time preferences in such a subjective dynamic discrete choice model. The empirical results we obtain from applying our model to adult smokers’ smoking decisions show that these subjective expectations data are indeed critical in understanding and predicting individual behavior under weaker assumptions than usually imposed in the literature.

Two of our main empirical results are particularly noteworthy. First, we find important differences when comparing adult smokers’ own subjective longevity expectations to the objective ones estimated using a rational expectations framework. Specifically, even though most subjective estimates of the effects of various survival determinants share the same signs as their objective counterparts, the relative weights attached to each determinant are quite different. Second, the utility and time preferences estimated using subjective expectations data show that individuals in our sample actually care more about their health, even if they choose to smoke, and are more forward-looking than is implied by a rational expectations framework.

With persistent unobserved heterogeneity introduced to the state transitions and preference parameters, we see that the potential endogeneity issue is indeed alleviated: agents within each type are more similar to each other in observed characteristics than to those in other groups, and agents in different groups exhibit different survival probabilities and utility and time preferences. The main message that subjective expectations data play an important role in our understanding of agents’ preferences and behaviors is also reinforced. Our counterfactual experiment further reveals that if adult smokers’ subjective expectations about their survival probabilities are set to be the same as the objective ones, the average smoking rate would be about 8 percentage points below its current level.

This difference between subjective and objective longevity expectations, whether it reflects valuable private information only contained in individuals’ subjective expectations or indeed results from respondents’ systematic mistakes, indicates that at least for the empirical setting in this paper, it is not innocuous to make the
simplifying yet strong rational expectations assumption about expectation formation. Subjective expectations data reported directly by survey respondents can help us better understand individuals’ preferences and behaviors.

Our empirical results are specific to the subjective questions and the particular sample used in this paper, so generalization should be considered with caution. However, the idea of incorporating subjective expectations data into a dynamic discrete choice model can be generalized readily to various dynamic decision-making processes, such as education choices, labor supply, and consumption and saving patterns.

Finally, the individuals analyzed in this paper are assumed to have mature opinions about the effects of different determinants on certain outcomes, even though these opinions are different from what we observe in the data. This assumption can be justified in the current empirical setting because these adult smokers arguably have passed the initial information-gathering stage and have given sufficient thought to their life expectancy. One interesting extension of our model would be to allow for learning or belief revision, which could play an important role in explaining behaviors in adolescents.

APPENDIX

A. Implications of Assumptions Concerning $\varepsilon_t$

Following Rust (1987), we make the following assumptions concerning the unobservable component in the preferences:

A1. Additive Separability: $u(s_t, a_t) = u(x_t, a_t) + \varepsilon_t(a_t)$.

A2. Conditional Independence:

$$p(x_{t+1}, \varepsilon_{t+1}(a_{t+1})|x_t, \varepsilon_t(a_t), a_t) = q(\varepsilon_{t+1}(a_{t+1})|x_{t+1})p(x_{t+1}|x_t, a_t)$$

$$q(\varepsilon_{t+1}(a_{t+1})|x_{t+1}) = q(\varepsilon).$$

A3. Extreme Value Error Distribution: $\varepsilon$ is i.i.d. with extreme value distribution.

Assumption A1 shows that the period utility function consists of two parts: $u(x_t, a_t)$, which depends only on the observed components, state variable $x$ and choice $a$ at time $t$; and the unobserved state variable $\varepsilon$.

Assumption A2 places simplifying restrictions on the transition probabilities by assuming that the transition of the unobserved state variable is independent of the observed state variables and the agent’s choice, and that the transition probabilities of the whole state space are multiplicatively separable in the observed and unobserved state variables, conditional upon the agent’s lagged choice and observed state variables. Furthermore, lagged unobserved state variables have no implications for the evolution of future state variables.

For example, Dominitz (1998) estimates respondents’ subjective probability distributions of weekly earnings using a sequence of subjective expectations questions.
The distribution of the unobserved state variable is difficult to identify without making strong parametric assumptions about its functional form. This distribution is therefore assumed to be known, with little loss of generality. The particular extreme value distribution in Assumption A3 guarantees a closed-form solution to the ex ante value function, also known in the literature as social surplus function (Rust 1994), as well as a convenient logistic functional form for the conditional choice probabilities.

Under Assumptions A1–A3, equations (2) and (3) together imply that the ex ante value function is related to the choice-specific value functions through the following expression:

\[(A1) \quad V(x_t) = E_\varepsilon \max_{a_t} \{V(x_t, a_t) + \varepsilon\}\]

\[= G(V(x_t, a_t), a_t = 0, \ldots, \#A) = \ln \left\{ \sum_{a_t \in A} \exp[V(x_t, a_t)] \right\},\]

where the first equality is obtained by replacing the first and third terms inside the square bracket on the right-hand side of equation (2) using the definition given in equation (3); while the second equality directly follows the extreme value error distribution of Assumption A3.35

B. Derivation of Equation (4)

Equation (A1) implies that the ex ante value function can be expressed as a function of choice specific value functions, which can be further written as a function of the differences between choice specific value functions \(D(x) \equiv V(x, 1) - V(x, 0)\) and one baseline choice specific value function.

With two choices, we can write equation (A1) in the following way:

\[(A2) \quad V(x_t) = \log\{\exp(V(x_t, 1)) + \exp(V(x_t, 0))\}\]

\[= \log\{\exp(V(x_t, 1) - V(x_t, 0)) + 1\} + V(x_t, 0)\]

\[= \log\{\exp(V(x_t, 0) - V(x_t, 1)) + 1\} + V(x_t, 1),\]

where the first equality is from the two-choice assumption, and the second and third equalities are obtained by adding and subtracting \(\ln(\exp(V(x_t, 0)))\) and \(\ln(\exp(V(x_t, 1)))\), respectively.

By inserting the second line of equation (A2) into equation (3) and rearranging the terms, we can have the following expression with \(D(x_t)\):

\[(A3) \quad V(x_t, 0) = \beta \int V(x_{t+1}, 0)p(x_{t+1}|x_t, 0) \, dx_{t+1}\]

\[= u(x_t, 0) + \beta \int \log(1 + e^{D(x_{t+1})})p(x_{t+1}|x_t, 0) \, dx_{t+1},\]

35 For a more detailed derivation of equation (A1), see Rust (1987).
where the second term on the right-hand side can be obtained directly from the data. Taking this term as given, the left-hand side subsequently defines a backward induction relation that recursively expresses \( V(x_t, 0) \) for all \( t \) as functions of \( u(x_t, 0) \) only.

Specifically, in the last period, the continuation value is equal to the instantaneous utility for all \( x \) and \( a \):

\[
(A4) \quad V(x_T, a_T) = u(x_T, a_T).
\]

For any period \( t < T \), we can rewrite equation (A3) and apply it recursively by moving the time index forward:

\[
V(x_t, 0) = u(x_t, 0) + \beta \int \log(1 + e^{D(x_{t+1})}) p(x_{t+1} | x_t, 0) \, dx_{t+1}
\]

\[
+ \beta \int V(x_{t+1}, 0) p(x_{t+1} | x_t, 0) \, dx_{t+1}
\]

\[
= u(x_t, 0) + \beta \int \log(1 + e^{D(x_{t+1})}) p(x_{t+1} | x_t, 0) \, dx_{t+1}
\]

\[
+ \beta \int u(x_{t+1}, 0) p(x_{t+1} | x_t, 0) \, dx_{t+1}
\]

\[
+ \beta^2 \int \int \log(1 + e^{D(x_{t+2})}) p_{t+1, t+2}(x_{t+2} | x_{t+1}, 0) p_{t+1}(x_{t+1} | x_t, 0) \, dx_{t+2} \, dx_{t+1}
\]

\[
+ \beta^2 \int \int V(x_{t+2}, 0) p_{t+1, t+2}(x_{t+2} | x_{t+1}, 0) p_{t+1}(x_{t+1} | x_t, 0) \, dx_{t+2} \, dx_{t+1}
\]

\[
= \ldots
\]

which can be (relatively more succinctly) written as

\[
V(x_t, 0) = u(x_t, 0) + \beta \int \log(1 + e^{D(x_{t+1})}) p(x_{t+1} | x_t, 0) \, dx_{t+1}
\]

\[
+ \beta \int V(x_{t+1}, 0) p(x_{t+1} | x_t, 0) \, dx_{t+1}
\]

\[
= u(x_t, 0) + \beta \int \log(1 + e^{D(x_{t+1})}) p(x_{t+1} | x_t, 0) \, dx_{t+1}
\]

\[
+ \beta \int u(x_{t+1}, 0) p(x_{t+1} | x_t, 0) \, dx_{t+1}
\]

\[
+ \beta^2 \int \log(1 + e^{D(x_{t+2})}) p_{t+1, t+2}(x_{t+2} | x_{t+1}, 0) \, dx_{t+2}
\]

\[
+ \beta^2 \int V(x_{t+2}, 0) p_{t+1, t+2}(x_{t+2} | x_{t+1}, 0) \, dx_{t+2}
\]

\[
= \ldots
\]
which further leads to the following backward induction relation:

\[(A5) \quad V(x_t, 0) = \sum_{s=t}^{T} \beta^{s-t} E[u(x_s, 0) | x_t = x, a = 0] \]

\[+ \sum_{s=t+1}^{T} \beta^{s-t} E[\log(1 + e^{D(x_s)}) | x_t = x, a = 0].\]

In the above conditional expectations, the distribution of \(x_s\) given \(x_t = x\) and \(a = 0\) is induced by the transition of the state variables \(x\) from \(t\) to \(s\) as if action \(a = 0\) is always taken between periods \(t\) to \(s - 1\).

Analogously, if we instead insert the third line of equation \((A2)\) into equation \((3)\), we have

\[(A6) \quad V(x_t, 1) - \beta \int V(x_{t+1}, 1)p(x_{t+1} | x_t, 1)dx_{t+1} \]

\[= u(x_t, 1) + \beta \int \log(1 + e^{-D(x_t+1)})p(x_{t+1} | x_t, 1)dx_{t+1}.\]

For any period \(t < T\), equation \((A6)\) translates into the following backward induction relation

\[(A7) \quad V(x_t, 1) = \sum_{s=t}^{T} \beta^{s-t} E[u(x_s, 1) | x_t = x, a = 1] \]

\[+ \sum_{s=t+1}^{T} \beta^{s-t} E[\log(1 + e^{-D(x_s)}) | x_t = x, a = 1].\]

In the above conditional expectations, the distribution of \(x_s\) given \(x_t = x\) and \(a = 1\) is induced by the transition of the state variables \(x\) from \(t\) to \(s\) as if action \(a = 1\) is always taken between periods \(t\) to \(s - 1\).

Given that the data identify \(D(x_t) \equiv V(x_t, 1) - V(x_t, 0)\), taking the difference between equations \((A5)\) and \((A7)\) and rearranging terms show that the difference

\[\sum_{s=t}^{T} \beta^{s-t} E[u(x_s, 1) | x_t = x, a = 1] - \sum_{s=t}^{T} \beta^{s-t} E[u(x_s, 0) | x_t = x, a = 0]\]

is identified through the data as

\[D(x_t) - \sum_{s=t+1}^{T} \beta^{s-t} E[\log(1 + e^{-D(x_s)}) | x_t = x, a = 1] \]

\[+ \sum_{s=t+1}^{T} \beta^{s-t} E[\log(1 + e^{D(x_s)}) | x_t = x, a = 0].\]
Given the linear form of the instantaneous period utility functions, this difference can be written as

\[
D(x_t) - \sum_{s=t+1}^{T} \beta^{s-t} E[\log(1 + e^{-D(x_s)}) | x_t = x, a = 1] \\
+ \sum_{s=t+1}^{T} \beta^{s-t} E[\log(1 + e^{D(x_s)}) | x_t = x, a = 0] \\
= \sum_{s=t}^{T} \beta^{s-t} E[x_{1s} | x_t = x, a = 1]' \theta_1 - \sum_{s=t}^{T} \beta^{s-t} E[x_{0s} | x_t = x, a = 0]' \theta_0,
\]

which is exactly equation (4).

As we discussed in Section I, the identification of the discount factor is based on Magnac and Thesmar (2002), who show that if there are certain exclusive restrictions that shift the expected future utilities (through, say, the transition of state variables) without entering individuals’ instantaneous utility functions, the discount factor can be identified. This somewhat abstract argument has intuitive appeal. Suppose two individuals are identical in all but one aspect. This difference affects their future state transitions, but not their instantaneous utilities. That is, this one different variable is exogenous to the utility function but still relevant to the state transitions. That makes this variable by definition the exclusive restriction. If these two individuals are completely myopic with a zero discount factor, then they should make the same choices whenever they are in the same states (except for the variable acting as the exclusive restriction), because their choices depend only on their identical instantaneous utilities. Systematic differences in their behaviors would not be expected. If these two individuals are not completely myopic, though, we would expect some differences in their behaviors because their state transitions, and therefore the total utilities associated with each choice, differ because of the exclusive restriction, even though they have the same instantaneous utilities. The more forward-looking they are, the greater the magnitude of this behavioral difference would be. Therefore, this relationship is used here to identify individuals’ time preference/discount factor.

C. Derivation of \( \sum_{s=t}^{T} \beta^{s-t} E[x_{as} | x_t = x, a]' \)

With discrete state variables, we can write the “regression” relation in equation (4) as

\[
(A8) \quad D(x_t) \quad - \quad \sum_{s=t+1}^{T} \beta^{s-t} P_{1s} \log(1 + e^{-D(x_s)}) \quad + \quad \sum_{s=t+1}^{T} \beta^{s-t} P_{0s} \log(1 + e^{D(x_s)}) \\
= \left[ \sum_{s=t}^{T} \beta^{s-t} P_{1s} x_1 \right]' \theta_1 \quad - \quad \left[ \sum_{s=t}^{T} \beta^{s-t} P_{0s} x_0 \right]' \theta_0,
\]
where \( D(x) \) denotes the vector of differences in the choice specific value functions with length equal to the number of discrete states, and \( P_{1,t}^{s} \) and \( P_{0,t}^{s} \) are defined as

\[
P_{1}^{t,s} = P_{1}^{t,t+1} P_{1}^{t+1,t+2} \ldots P_{1}^{s-1,s}
\]

\[
P_{0}^{t,s} = P_{0}^{t,t+1} P_{0}^{t+1,t+2} \ldots P_{0}^{s-1,s}
\]

with

\[
P_{1}^{t,t} = I
\]

\[
P_{0}^{t,t} = I.
\]

More precisely, we can write

\[
\sum_{s=t}^{T} \beta^{s-t} P_{1}^{t,s} = I + \beta P_{1}^{t,t+1} + \beta^{2} P_{1}^{t,t+2} + \ldots + \beta^{T-t} P_{1}^{t,T}
\]

\[
= I + \beta P_{1}^{t,t+1} + \beta P_{1}^{t,t+1} \beta P_{1}^{t+1,t+2} + \ldots + \beta P_{1}^{t,t+1} \times \ldots \times \beta P_{1}^{T-1,T}.
\]

Note that when death is allowed and the states in \( P_{1} \) and \( P_{0} \) do not include death, \( P_{1} \) and \( P_{0} \) only have to be “sub-”stochastic matrices, in the sense that their rows sum to \( < 1 \) instead of \( 1 \).

If we define

\[
Q_{t} = \sum_{s=t}^{T} \beta^{s-t} P_{1}^{t,s},
\]

then we can compute \( Q_{t} \) through recursive relations.

First of all,

\[
Q_{T} = I;
\]

and then for all \( t < T \)

\[
Q_{t} = \beta P_{1}^{t,t+1} Q_{t+1} + I.
\]

Also note that there is a certain relation between the matrices on the left- and right-hand side of equation (A8). In particular, the left-hand side matrices are \( \beta P_{1}^{t,t+1} \) and \( \beta P_{0}^{t,t+1} \) multiplied by the right-hand side matrices shifted forward by one period

\[
\sum_{s=t+1}^{T} \beta^{s-t} P_{1}^{t,s} = \beta P_{1}^{t,t+1} \left( \sum_{s=(t+1)}^{T} \beta^{s-(t+1)} P_{1}^{(t+1),s} \right),
\]

and

\[
\sum_{s=t+1}^{T} \beta^{s-t} P_{0}^{t,s} = \beta P_{0}^{t,t+1} \left( \sum_{s=(t+1)}^{T} \beta^{s-(t+1)} P_{0}^{(t+1),s} \right).
\]
REFERENCES


